Solving Transient Eddy Current Problems with Radial Basis Function Method in Frequency Domain

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Abstract — The Radial Basis Function (RBF) method in frequency domain is proposed in this paper for solving transient eddy problems. The method combines the RBF collocation method with the technique of processing the time variable in frequency domain. The RBF method is utilized to solve the frequency model so as to obtain the approximate solution in frequency domain. While the Fourier transform realizes the time and frequency transformation. On the basis of proper time-frequency parameter settings and the mono-frequency harmony field solution with the RBF method, the paper gives the implementation scheme of RBF method in frequency domain. The numerical simulation of the transient magnetic field in the aluminum flake board showed that the method suggested in this paper is feasible and effective. In addition, the method can maintain good precision under large time and space interval because of the RBF's good approaches ability.

I. INTRODUCTION

The transient eddy current field calculation is of great significance for product design and performance analysis. In recent years, meshless methods like EFGM (Element Free Galerkin Method), RBF method and RKPM (Reproducing Kernel Particle Method) are gradually applied into transient eddy current field calculation. S.A.Viana[1] employed the local radial point interpolation method. K.R.Shao introduced the radial function into boundary element[2~3]. Zhang.Y[4]. utilizes multi-quadrics collocation method in time domain. Compared to EFGM， the RBF method takes great advantages of approximate function, calculation model and boundary processing. However, time domain. method relies greatly on step size. Frequency is an important parameter especially for system frequency response analysis. Thus, this paper proposes frequency RBF method for analyzing transient eddy current problems.

First, the transient eddy current model and RBF collocation approximation theory will be given in section II. Then the principles of frequency RBF method will be given in section III. In section IV the frequency RBF simulation of thin aluminum plate transient magnetic field is present.

II. TRANSIENT EDDY CURRENT MODEL AND RBF COLLOCATION APPROXIMATION THEORY

In transient eddy problems, the quasi-static magnetic field model is built. Generally, the initial boundary value form of two-dimension problems is:

$$\begin{cases} \nabla^2 u(t,\boldsymbol{x}) = \mu\sigma \dfrac{\partial u(t,\boldsymbol{x})}{\partial t} & , \quad \boldsymbol{x}\in\Omega, 0<t<T \\ u(t,\boldsymbol{x}) = \bar{u}(t,\boldsymbol{x}) & , \quad \boldsymbol{x}\in\Gamma_{\mathrm{u}}, 0<t<T \\ \dfrac{\partial u(t,\boldsymbol{x})}{\partial n} = \bar{q}(t,\boldsymbol{x}) & , \quad \boldsymbol{x}\in\Gamma_{\mathrm{q}}, 0<t<T \\ u(0,\boldsymbol{x}) = u_0(\boldsymbol{x}) & , \quad \boldsymbol{x}\in\Omega\cup\Gamma \end{cases} \tag{1}$$

where u is magnetic field intensity or magnetic vector potential component in rectangular coordinate system. Ω is the solution domain; Γ_{u}, Γ_{q} are respectively the first and second-type boundary conditions; T is the time interval to be investigated.

The RBF is the continuous real valued function of radial vector. The adopted RBF in this paper is multi-quadric (Abbreviated as. MQ) function, its expression in three-dimension is:

$$\phi(\boldsymbol{x}) = \sqrt{(x-c_x)^2 + (y-c_y)^2 + (z-c_z)^2 + \alpha^2} \tag{2}$$

where (c_x, c_y, c_z) is the center of the basis function; $\alpha = \beta\|\boldsymbol{c}_i - \boldsymbol{c}_j\|$ is shape parameter. Considering an unknown function f and its discrete data nodes, the RBF interpolation is linear combination of RBF. When function centers are chosen on the interpolation points, we get the following formula[5]:

$$f(\boldsymbol{x}_i) = \sum_{j=1}^{N} \lambda_j \phi(\|\boldsymbol{x}_i - \boldsymbol{x}_j\|),\ i = 1,2,\dots,N \tag{3}$$

with the matrix form:

$$\begin{bmatrix} \phi(\|\boldsymbol{x}_1-\boldsymbol{x}_1\|) & \phi(\|\boldsymbol{x}_1-\boldsymbol{x}_2\|) & \cdots & \phi(\|\boldsymbol{x}_1-\boldsymbol{x}_N\|) \\ \phi(\|\boldsymbol{x}_2-\boldsymbol{x}_1\|) & \phi(\|\boldsymbol{x}_2-\boldsymbol{x}_2\|) & \cdots & \phi(\|\boldsymbol{x}_2-\boldsymbol{x}_N\|) \\ \vdots & \vdots & \vdots & \vdots \\ \phi(\|\boldsymbol{x}_N-\boldsymbol{x}_1\|) & \phi(\|\boldsymbol{x}_N-\boldsymbol{x}_2\|) & \cdots & \phi(\|\boldsymbol{x}_N-\boldsymbol{x}_N\|) \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \end{bmatrix} = \begin{bmatrix} f(\boldsymbol{x}_1) \\ f(\boldsymbol{x}_2) \\ \vdots \\ f(\boldsymbol{x}_N) \end{bmatrix} \tag{4}$$

Thus the weight coefficients are available:

$$\boldsymbol{\lambda} = [\boldsymbol{\Phi}]^{-1}\boldsymbol{f} \tag{5}$$

where $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \dots, \lambda_N]^{\mathrm{T}}$ and $\boldsymbol{f} = [f(\boldsymbol{x}_1), f(\boldsymbol{x}_2), \dots, f(\boldsymbol{x}_N)]^{\mathrm{T}}$. Therefore, the function $f(\boldsymbol{x})$ at any point $\boldsymbol{x}$ is:

$$f(\boldsymbol{x}) = \boldsymbol{\Phi}(\boldsymbol{x})\boldsymbol{\lambda} = \boldsymbol{\Phi}(\boldsymbol{x})[\boldsymbol{\Phi}]^{-1}\boldsymbol{f} \tag{6}$$

III. RBF METHOD IN FREQUENCY DOMAIN

In the time modal Eq.(1), function $u(t,\boldsymbol{x})$ consists of time and space variables with first and second order partial derivatives So in frequency domain, the above modal will be transformed at first, then get the frequency domain solution, and finally we can acquire the time domain solution through inverse transformation.

The corresponding frequency domain model is:

$$\begin{cases} \nabla^2 U(\omega,\boldsymbol{x}) = j\omega\mu\sigma U(\omega,\boldsymbol{x}), \boldsymbol{x}\in\Omega, -\infty<\omega<\infty \\ U(\omega,\boldsymbol{x}) = \overline{u}(\omega,\boldsymbol{x}), \boldsymbol{x}\in\Gamma_{\mathrm{u}}, -\infty<\omega<\infty \\ \dfrac{\partial U(\omega,\boldsymbol{x})}{\partial n} = \overline{q}(\omega,\boldsymbol{x}), \boldsymbol{x}\in\Gamma_{\mathrm{q}}, -\infty<\omega<\infty \\ \dfrac{1}{2\pi}\int_{-\infty}^{\infty} U(\omega,\boldsymbol{x})\mathrm{d}\omega = u_0(\boldsymbol{x}), \boldsymbol{x}\in\Omega\cup\Gamma \end{cases} \tag{11}$$

where $U(\omega,\boldsymbol{x})$ is the Fourier transform function of $u(t,\boldsymbol{x})$. The implementation scheme of frequency domain RBF method is as follows:

A. Time and Frequency parameter settings

In the Fourier transformation, the truncating and discretizing for time and frequency functions $\overline{q}(t,\boldsymbol{x})$, $u(t,\boldsymbol{x})$, $\overline{u}(\omega,\boldsymbol{x})$ and $U(\omega,\boldsymbol{x})$ are inevitable. According to signal theory, the time step should meet $\Delta t \le \pi/\omega_c$ where ω_c is maximum frequency. The discrete approximate functions are $\overline{q}(n\Delta t,\boldsymbol{x})$, $u(n\Delta t,\boldsymbol{x})$, $\overline{u}(k\Delta\omega,\boldsymbol{x})$ and $U(k\Delta\omega,\boldsymbol{x})$.

B. RBF parameter settings

The RBF parameters include center $\boldsymbol{c}_i$, shape parameter β_i and collocation points. Assume the number of RBFs and collocation points are respectively N and $M=M_1+M_2$ (where M_1 in domain and M_2 on the boundary).

C. Calculation of Frequency Solution

It mainly includes following steps:

(1) Component frequency domain solution at single frequency. Assume the frequency point is $\omega=\omega_0$ ($k=k_0$), solving the corresponding discrete Helmholtz equations in phasor form:

$$\begin{cases} \nabla^2 U(k_0\Delta\omega,\boldsymbol{x}_i) = jk_0\Delta\omega\mu\sigma U(k_0\Delta\omega,\boldsymbol{x}_i), \boldsymbol{x}_i\in\Omega \\ U(k_0\Delta\omega,\boldsymbol{x}_b) = \overline{u}(k_0\Delta\omega,\boldsymbol{x}_b), \boldsymbol{x}_b\in\Gamma_{\mathrm{u}} \\ \dfrac{\partial U(k_0\Delta\omega,\boldsymbol{x}_b)}{\partial n} = \overline{q}(k_0\Delta\omega,\boldsymbol{x}_b), \boldsymbol{x}_b\in\Gamma_{\mathrm{q}} \end{cases} \tag{12}$$

The matrix form is:

$$\begin{bmatrix} \boldsymbol{L}[\phi(\boldsymbol{x}_i)] \\ \boldsymbol{B}[\phi(\boldsymbol{x}_b)] \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\lambda}}^{(k_0)} \end{bmatrix} = \begin{bmatrix} jk_0\Delta\omega K U(k_0\Delta\omega,\boldsymbol{x}_i) \\ G(k_0\Delta\omega,\boldsymbol{x}_b) \end{bmatrix} \tag{13}$$

where $\boldsymbol{L}$ and $\boldsymbol{B}$ means domain and boundary operator. So we get the coefficient $\dot{\boldsymbol{\lambda}}^{(k_0)}$ and approximate solution: $U(k_0\Delta\omega,\boldsymbol{x}) = \sum_i \dot{\lambda}_i^{(k_0)}\phi_i\left(\|\boldsymbol{x}-\boldsymbol{c}_i\|,\alpha_i\right)$.

(2) Frequency sweeping process, with step $\Delta\omega$ in range $-\omega_c<\omega<\omega_c$, find every frequency point solution and corresponding coefficients $\dot{\boldsymbol{\lambda}}^{(k_i)}$ similarly.

(3) Combining component frequency solution into frequency domain solution, we obtain approximate solution $U(\boldsymbol{k}\Delta\omega,\boldsymbol{x}) = \sum_i \dot{\lambda}_i^{(\boldsymbol{k})}\phi_i\left(\|\boldsymbol{x}-\boldsymbol{c}_i\|,\alpha_i\right)$.

D. Time Domain Solution

Time domain approximate solution $u(t,\boldsymbol{x})$ can be calculated from the frequency solution. With inverse Fourier transformation, we get the time solution: $u(t,\boldsymbol{x}) = \dfrac{1}{2\pi}\sum_k U(k\Delta\omega,\boldsymbol{x})e^{jk\Delta\omega t}\Delta\omega$.

IV. SIMULATION OF THIN ALUMINUM PLATE

Considering long thin wire wrapped aluminum plate with thickness of d (In positive x axis direction), we will analyze the magnetic field changing process with rectangular impulse excitation. The mathematical model is:

$$\begin{cases} \dfrac{\partial^2 H_z}{\partial x^2} = \mu\sigma\dfrac{\partial H_z}{\partial t}, & -0.5d<x<0.5d, 0<t<T \\ H_z(0,x)=0, & -0.5d<x<0.5d \\ H_z(t,\pm 0.5d)=H_0, & 0\le t\le\tau \\ H_z(t,\pm 0.5d)=0, & \tau<t\le T \end{cases} \tag{14}$$

The above parameter are d=1.0cm, T=20.0s, τ =2.0s, $\mu=4\pi\times10^{-7}$ (H/m), $\sigma=3.82\times10^{7}$ (S/m) and $H_0=1.0$A/m. So the discrete step is $\Delta t=0.05$s and $\Delta\omega=0.05\times2\pi$ rad/s. The RBF centers and collocations points are equally locate in [-0.5cm,0.5cm] with step h=0.05cm. Select $\beta=5.0$ for shape parameter. The simulation results are:

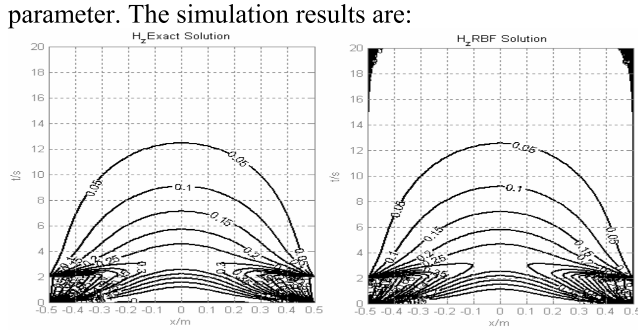

Fig.1 Exact and RBF solutions

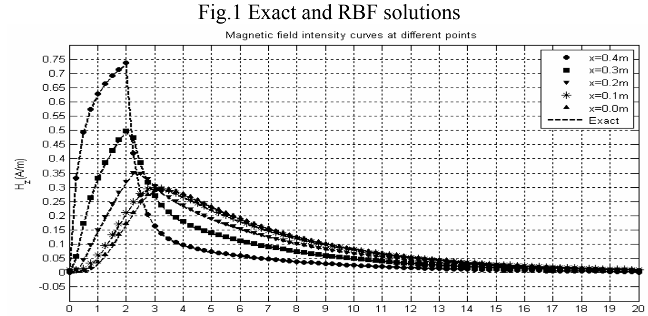

Fig.2 RBF solutions at selected points for magnetic field intensity

V. REFERENCES

[1] S.A.Viana, D.Rodger, H.C.Lai. Application of the local radial point interpolation method to solve eddy-current problems[J] IEEE Transactions on Magnetics, 2006, 42(4): 91-94.

[2] Yong Zhang, K.R.Shao, Youguang Guo, Jianguo Zhu, D.X.Xie, J.D.lavers. A comparison of point interpolative boundary meshless method based on PBF and RBF for transient eddy-current analysis[J] IEEE Transactions on Magnetics, 2007, 43(4): 1497-1500

[3] Yong Zhang, Shao, K.R. Youguang Guo Lavers, J.D. A boundary meshless method for transient eddy current problems[J] IEEE Transactions on Magnetics, 2005, 41(10): 4090- 4092

[4] Zhang, Y. Shao, K.R. Guo, Y. Zhu, J. Lavers, J.D. Multiquadrics Collocation Method for Transient Eddy Current Problems[J] IEEE Transactions on Magnetics, 2006, 42(10): 3183- 3185

[5] E.E. Hart,, S.J.Cox, K.Djidjeli, V.O.Kubytskyi. Solving an eigen value problem with aperiodic domain using radial basis functions[J]. Engineering Analysis with Boundary Elements,2009(33): 258-262.